УДК 624.012

ВИЗНАЧЕННЯ ПРОГИНІВ ЗАЛІЗОБЕТОННИХ РАМ П-ПОДІБНОЇ ФОРМИ

DETERMINATION OF DEFLECTIONS OF U-SHAPED REINFORCED CONCRETE FRAMES

Бабич В.Є, к.т.н., доц., (Національний університет водного господарства та природокористування, м. Рівне), ORCID.ORG 0000-0003-2811-2423 **Гомон П.С.,** к.т.н., доц., (Національний університет водного господарства та природокористування, м. Рівне), ORCID.ORG /0000-0002-5312-0351

Babych V.Y., Ph.D. (Tech.) assoc. prof., (National University of Water and Environmental Engineering, Rivne), ORCID.ORG 0000-0003-2811-2423 **Gomon P.S., Ph.D.** (Tech.), assoc. prof., (National University of Water and Environmental Engineering, Rivne), ORCID.ORG /0000-0002-5312-0351

У статті розглянуто методику розрахунку прогинів залізобетонних балок за граничними станами другої групи відповідно до сучасних нормативних вимог. Визначено основні залежності для обчислення кривизни елемента в станах «без тріщин» та «із тріщинами» з урахуванням осьових деформаційних характеристик матеріалу. Запропоновано алгоритм розрахунку, що включає визначення граничних деформацій бетону, моменту тріщиноутворення, напружень у розтягнутій арматурі та висоти стиснутої зони бетону. Методика враховує вплив тривалості навантаження, циклічних впливів і жорсткісних характеристик елементів. Представлено ітераційний підхід до уточнення параметрів згинального моменту, який дозволяє підвищити точність визначення кривизни та прогину. Отримані результати можуть бути використані для проектування залізобетонних конструкцій із забезпеченням нормативної жорсткості та експлуатаційної надійності.

This paper presents a methodology for calculating the deflections of reinforced concrete u-shaped frames designed according to the second group of limit states, based on the consideration of deformation characteristics of the material and external loading conditions. The deflections in u-shaped frames primarily depend on bending deformations, which can be determined by calculating the curvature. The curvature of the element is expressed through a formula that accounts for the impact of cracks on the stiffness of the material. The proposed algorithm includes several stages for determining the curvature in the "crack-free" and "cracked" states. These stages involve the calculation of the cracking moment, the internal stresses in the reinforcement, and the deformations in the concrete under tension and compression.

The methodology focuses on calculating the height of the compressed concrete zone, which is essential for ensuring the internal force equilibrium in the cross-section. The algorithm also involves an iterative process for refining the values of internal stresses and bending moments, ensuring accurate prediction of u-shaped frames deflections. The importance of determining the stiffness reduction factor, which accounts for the loss of stiffness due to cracks and material deformations, is emphasized in the paper. Additionally, the paper explores the effect of loading duration and repeated loads on the average deformation behavior of the u-shaped frame.
The methodology is based on the assumption of plane cross-sections and incorporates the use of polynomials to represent the stress-strain relationships for concrete in both tension and compression. The calculation procedure involves determining the initial deformation in the concrete, calculating the stresses in the reinforcement, and then calculating the cracking moment, followed by the deflection calculation using the generalized curvature.
The method also accounts for external loading conditions, including both short-term and long-term loads, by considering the influence of repeated load cycles and the duration of loading on the material's behavior. The paper provides a detailed explanation of how to use these methods in practical applications, including the necessary formulas and steps for accurate deflection predictions. The proposed methodology ensures the reliable design of reinforced concrete structures, taking into account all relevant factors such as material properties, structural dimensions, and loading conditions. The calculations are consistent with modern standards, such as Eurocode, and can be applied to various types of reinforced concrete structures.

Ключові слова:
Залізобетонні балки, прогини конструкцій, граничні стани, тріщиноутворення..
Reinforced concrete u-shaped frames, deflections of structures, limit states, crack formation.

Introduction. Reinforced concrete U-shaped frames are widely used in the construction of various buildings and structures. In new construction, they are planned for use in protective civil defense structures. Frames are considered statically indeterminate systems, the static analysis of which is currently performed as elastic systems. In real operating conditions, due to the manifestation of the plastic properties of concrete and reinforcement, the determined forces may be redistributed between the normal sections of the frame elements, deviating from the values calculated for the frames as elastic systems.

Regulatory documents recommend calculating statically indeterminate reinforced concrete structures taking such a process into account [1-3]. The redistribution of forces in statically indeterminate reinforced concrete structures has been studied for several decades [4-5], but a perfect calculation methodology for frames does not yet exist. Therefore, it is relevant to investigate the stress-strain state of the sections of frame elements calculated using two different methodologies and to determine their advantages and disadvantages.

The design of reinforced concrete structures based on limit states is a crucial aspect of modern construction. One of the primary tasks in structural design is the accurate determination of deflections, which depend on the deformation characteristics of the material and external loads. For reinforced concrete u-shaped frames, deflections are predominantly governed by bending deformations [6]. Accurate deflection calculations require a detailed analysis of the structural behavior both before and after the formation of cracks. This paper proposes a methodology for determining u-shaped frame curvature and evaluating deflections that takes into account the nonlinear properties of the material, crack formation, and the dynamic nature of loading. The application of these methods ensures reliable analysis and design of reinforced concrete structures in accordance with modern standards, such as the Eurocode [7].

The fundamental provisions for the design of reinforced concrete structures according to the serviceability limit states (second group of limit states) involve the determination of u-shaped frame deflections based on the principles of structural mechanics. These deflections are evaluated as a function of the axial deformation (stiffness) characteristics of the reinforced concrete element at various cross-sections along its length.

Since u-shaped frame deflections are primarily influenced by bending deformations, they can be determined through curvature analysis. Based on the general expression for deformation characteristics, the curvature of u-shaped frams can be evaluated using the following formula [2].

$$\chi = \xi\chi_{II} + (1-\xi)\chi_I \, , \tag{1}$$

χ – curvature of the element;

χ_{II} , χ_I – curvatures determined for the section state corresponding to the "uncracked" and "cracked" conditions;

ξ – a coefficient of stiffness reduction in the tension zone of the cross-section, which is determined by the following expression.

$$\xi = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 ; \tag{2}$$

β – a coefficient that accounts for the influence of load duration or repeated loading on the average deformation under load (β = 1 for a single short-term load; β = 0,5 for sustained loads or multiple loading cycles);

σ_s - stress in the tensile reinforcement;

σ_{sr} - stress in the tensile reinforcement under loading conditions that cause the formation of the first crack.

In expression (2) the ratio σ_s/σ_{sr} may be replaced by the ratio M/M_{cr}, where M - is the acting moment in the section from the external operational load, and M_{cr} – is the cracking moment.

The guidelines do not provide a specific methodology for determining σ_{sr} or M_{cr}, but in the fundamental provisions, only conditions are given for this. In view of the above, it is crucial to develop a methodology and calculation algorithm for reinforced concrete u-shaped frames based on deformations for practical use of the fundamental provisions. These issues are addressed in this paper.

Determination of stresses and bending moments σ_{sr} and M_{cr}. The design of reinforced concrete structures according to the second group of limit states is largely based on Eurocode 2. The stresses in the reinforcement σ_{sr} and the cracking moment can be determined under the condition that the tensile strains in the concrete reach the limiting value, equal to...

$$\varepsilon_{ctu} = -2f_{ctm}/E_{ck}, \tag{3}$$

where ε_{ctu} - limiting tensile strains in the concrete;

f_{ctm} - characteristic value of the concrete tensile strength in axial tension;

E_{ck} - characteristic value of the initial modulus of elasticity of concrete.

Let us consider the methodology for calculating a reinforced concrete u-shaped frame with a rectangular cross-section and single reinforcement (Fig. 1). Considering the "uncracked" section state and using the plane section hypothesis (Fig. 1a), the following expression can be written:

$$\varepsilon_{ctII} = \varepsilon_{sII} = \frac{\varepsilon_{stu}}{h-x}(h-x-a); \tag{4}$$

$$\varepsilon_{cII} = \frac{\varepsilon_{stu}}{h-x}x, \tag{5}$$

where x –hight of compressed zone.

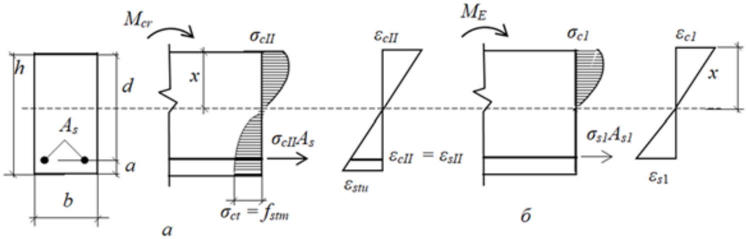

Fig. 1. Stressed-deformed state of the rectangular cross-section: a – in the "uncracked" limit state, b – in the "cracked" state.

By determining $\varepsilon_{sII} = \varepsilon_{cr}$ and ε_{cII} , , the stress in the tensile reinforcement before cracking σ_{sr} , the cracking moment M_{cr}, and the curvature χ_{II}. To do this, it is necessary to determine the height of the compressed concrete zone x, based on the equilibrium condition of internal forces in the section, i.e., from the condition

$$S_{cII} = S_{ctII} + S_{sII} , \tag{6}$$

where S_{cII} , S_{ctII} , S_{sII} - accordingly, the internal forces in the compressed and tensile zones of the concrete, as well as in the tensile reinforcement, are considered.

Using the relationship between stresses and strains in concrete under compression and tension in the form of a fifth-degree polynomial [1], condition (6) can be written as [5]:

$$f_{ck} b z_1 \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_{cII}}{\varepsilon_{c1}} \right)^k = f_{ctm} b (h - z_1) \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_{ctII}}{\varepsilon_{ctu}} \right)^k + E_s A_s \frac{\varepsilon_{stu}}{h - z_1} (h - z_1 - a) \tag{7}$$

The coefficients of the relationship "$\sigma_c - \varepsilon_c$" are taken according to [1] for the second group of limit states. In equation (7) $z_1 = x$ represents a fixed value of the height of the compressed zone, which is initially specified. If equality (7) is satisfied with a given accuracy m, which can be taken as $\pm 2\%$ or $\pm 5\%$, it can be assumed that the values of ε_{cII} and ε_{sII} are determined. If equality (7) is not satisfied, the value of z_1 , must be adjusted, and the new values of ε_{cII} and ε_{sII} should be determined using formulas (4) and (5), and equation (7) should be checked again. In the first approximation, $\pm i z_1$, where i – is the assumed step change in z_1 (typically $i = 0{,}05$).

Using the finally determined values of z_1, ε_{cII} and ε_{sII} the stress in the reinforcement before the formation of cracks can be determined.

$$\sigma_{sr} = \varepsilon_{sII} E_s ; \tag{8}$$

the cracking moment M_{cr} can be determined using the formula [5]:

$$M_{cr} = f_{ck} b z_1^2 \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_{cII}}{\varepsilon_{c1}} \right)^k + f_{ctm} b (h - z_1)^2 \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_{ctII}}{\varepsilon_{ctu}} \right)^k + E_s A_s \frac{\varepsilon_{stu}}{h - z_1} (h - z_1 - a)^2 \tag{9}$$

The curvature of the u-shaped frame in the "uncracked" state can also be determined.

$$\chi_{II} = \frac{\varepsilon_{cII} + \varepsilon_{sII}}{d} . \tag{10}$$

Determination of the curvature of the u-shaped frame χ_I in the "cracked" section state. The value of χ_I is determined under the action of the operational load, which induces a bending moment M_{Ee} in the section. The section is considered in the "cracked" state (Fig. 1b). In this state, the unknowns are the height of the compressed concrete zone *x*, the strain in the compressed concrete ε_{cI} and the strain in the tensile reinforcement ε_{sI}. This problem can be solved as follows.

By specifying the stepwise change in the strain of the compressed concrete zone ε_c, using the iteration method, the corresponding values of the strain in the tensile reinforcement ε_s and the bending moments *M*, that correspond to the given strains can be found. For example, the strains in the concrete can be varied from 0 to ε_{c1} in steps of $0{,}1\varepsilon_{c1}$. Based on the obtained data, graphs of the relationships between M and ε_c, as well M and ε_s, can be plotted. From the obtained graphs, ε_{c1} and ε_{s1} under the action of the external moment $M = M_{Ee}$ can be determined either analytically or graphically.

For fixed values of ε_c and $z_1 = x$, the bending moment of internal forces can be determined using the formula [5].

$$M_S = f_{ck} b z_1^2 \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^k + \varepsilon_s E_s A_s (d - z_1) \tag{11}$$

at the same time, the condition must be satisfied:

$$f_{ck} b z_1 \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^k = \varepsilon_s E_s A_s\ . \tag{12}$$

For a given ε_c, the problem is solved at each stage using the method of successive approximation of the values of *z* until the condition (12) is satisfied with the specified accuracy.

In formulas (11) and (12), the values of ε_s are determined using:

$$\varepsilon_s = \frac{\varepsilon_c}{z_1} (d - z_1)\ . \tag{13}$$

The stress in the reinforcement σ_s under the action of the operational moment is determined using the following formula:

$$\sigma_{s1} = \varepsilon_{s1} E_s, \tag{14}$$

the curvature of the u-shaped frame in the "cracked" state χ_I is determined using the following formula:

$$\chi_1 = \frac{\varepsilon_{c1} + \varepsilon_{s1}}{d}\ . \tag{15}$$

Determination of the u-shaped frame deflection. The generalized curvature of the u-shaped frame χ It is determined using formula (1), taking into account formula (2).

The u-shaped frame deflection is calculated using the construction mechanics formula in the form of:

$$f = s\chi l^2 , \qquad (16)$$

where l - the effective span of the u-shaped frame;

s - the coefficient that depends on the type of u-shaped frame support and the nature of the load.

Conclusions:

1. The methodology presented in the article allows for the precise determination of deflections in reinforced concrete u-shaped frame s, taking into account the effect of cracks on the material stiffness and the changes in its deformation characteristics. This ensures a more accurate and reliable approach to the design of reinforced concrete structures for second-group limit states.
2. The use of an iterative method for refining parameters, such as deformations and moments, helps improve the accuracy of deflection and cracking moment calculations. This approach provides more reliable results for the real operating conditions of structures.
3. The developed methodology is useful for practical application in the design of reinforced concrete structures, as it considers important factors such as cyclic loading, load duration, and changes in material properties. This ensures compliance with modern standards, including Eurocodes, and guarantees the safety and reliability of structures.

1. DBN V.2.6-162:2017. Betonni ta zalizobetonni konstruktsii. Osnovni polozhennia [Chynnyi vid 2018-01-01]. – K. : Minrehion Ukrainy, 2017. – 49 s. – (Derzhavni budivelni normy Ukrainy).

2. DSTU-N B V.2.6-212:2016. Nastanova z proiektuvannia betonnykh ta zalizobetonnykh konstruktsii z vazhkoho betonu za Yevrokodom 2. – K. : Minrehion Ukrainy, 2016. – 96 s. – (Natsionalni nastanovy Ukrainy).

3. Bambura A.M. Proektuvannia betonnykh i zalizobetonnykh konstruktsii za natsionalnymy normatyvnymy dokumentamy DBN V.2.6-98:2009 ta DSTU B V.2.6-156:2010 / Bambura A.M., Nemchynov Yu.I., Hurkivskyi O.B., Bezbozhna M.S., Dorohova O.V., Sazonova I.R., Ivlieva N.P., Barashykov A.Ia.// Budivelni konstruktsii: Zbirnyk naukovykh prats. – Kyiv: DP NDIBK, 2011. – Vypusk 74. – Knyha 1. – s.10 – 19.

4. Babych Ye.M Rozrakhunok nerozriznykh zalizobetonnykh balok iz vykorystanniam deformatsiinoi modeli: Rekomendatsii / Babych Ye.M., Babych V.Ie., Savytskyi V.V. – Rivne: NUVHP, 2005. – 38 s.

5. Homon P.S. Pobudova epiury napruzhen stysnutoi zony betonu dlia normalnykh pereriziv zghynalnykh zalizobetonnykh elementiv pislia dii malotsyklovykh navantazhen z vykorystanniam deformatsiinoi modeli / Homon P.S., Homon S.S. / Resursoekonomichni materialy, konstruktsii, budivli i sporudy: Zbirnyk naukovykh prats. – Rivne: NUVHP, 2009. – Vypusk 19.- S.124-130.

6. Homon P.S Vyznachennia prohynu zalizobetonnoi balky z vykorystanniam diahram deformuvannia betonu ta armatury / Homon P.S., Bychkovskyi T.O. / Resursoekonomni materialy, konstruktsii, budivli i sporudy: Zbirnyk naukovykh prats. – Rivne: NUVHP, 2013. – Vypusk 27. - S.56-61.

7. Eurocode 2: Design of Concrete Structures. EN 1992 – 1.1: General Rules and Rules for Buildings. – Brussels: CEN, 2004. – 226 p.